Published by Institute of Physics Publishing for SISSA/ISAS
Received: *August 4, 2003*
Accepted: *February 4, 2004*

Nilpotent deformations of $N = 2$ superspace

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Abstract: We investigate deformations of four-dimensional $N = (1,1)$ euclidean superspace induced by nonanticommuting fermionic coordinates. We essentially use the harmonic superspace approach and consider nilpotent bi-differential Poisson operators only. One variant of such deformations (termed chiral nilpotent) directly generalizes the recently studied chiral deformation of $N = (\frac{1}{2}, \frac{1}{2})$ superspace. It preserves chirality and harmonic analyticity but generically breaks $N = (1,1)$ to $N = (1,0)$ supersymmetry. Yet, for degenerate choices of the constant deformation matrix $N = (1, \frac{1}{2})$ supersymmetry can be retained, i.e. a fraction of 3/4. An alternative version (termed analytic nilpotent) imposes minimal nonanticommutativity on the analytic coordinates of harmonic superspace. It does not affect the analytic subspace and respects all supersymmetries, at the expense of chirality however. For a chiral nilpotent deformation, we present non(anti)commutative euclidean analogs of $N = 2$ Maxwell and hypermultiplet off-shell actions.

Keywords: Extended Supersymmetry, Superspaces, Non-Commutative Geometry.

Contents

1. Introduction

Moyal-type deformations of superfield theories are currently a subject of intense study (see, e.g. [1]–[5]). Analogous to noncommutative field theories on bosonic spacetime, noncommutative superfield theories can be formulated in ordinary superspace by multiplying functions given on it via a star product which is generated by some bi-differential operator or Poisson structure P. The latter tells us directly which symmetries of the undeformed (local) field theory are explicitly broken in the deformed (nonlocal) case.

Generic Moyal-type deformations of a superspace are characterized by a constant graded-antisymmetric non(anti)commutativity matrix (C^{AB}).[1] A minimal deformation of euclidean $N = 1$ superspace — more suitably denoted as $N = (\frac{1}{2}, \frac{1}{2})$ superspace — was considered in a recent paper [8]. For the chiral $N = 1$ coordinates $(x_L^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ the non(anti)commutativity was restricted to

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} := \theta^\alpha \star \theta^\beta + \theta^\beta \star \theta^\alpha = C^{\alpha\beta}\,, \tag{1.1}$$

i.e. the basic star products read

$$\theta^\alpha \star \theta^\beta = \theta^\alpha\,\theta^\beta + \frac{1}{2}C^{\alpha\beta}\,, \quad \theta^\alpha \star x_L^m = \theta^\alpha\,x_L^m\,, \quad x_L^m \star x_L^n = x_L^m\,x_L^n\,, \tag{1.2}$$

with $(C^{\alpha\beta})$ being some constant symmetric matrix. Note that the bosonic and the antichiral coordinates have undeformed commutation relations with everyone, so (C^{AB}) is rather degenerate here. For functions A and B of $(x_L^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ the star product (1.2) is generated as

$$A \star B = A\,\mathrm{e}^P B = A\,B + A\,P\,B + \frac{1}{2}A\,P^2 B \tag{1.3}$$

[1] The interpretation of quantizing fermionic systems as a deformation of the Grassmann algebra, in analogy with the Weyl-Moyal quantization of bosonic systems, can be traced back to [6, 7].

where the bi-differential operator

$$P = -\frac{1}{2}\overleftarrow{\partial}_\alpha C^{\alpha\beta}\overrightarrow{\partial}_\beta \qquad \text{is nilpotent:} \qquad P^3 = 0\,. \tag{1.4}$$

This defines a particular case of a deformed superspace. It retains $N = (1/2, 0)$ of the original $N = (1/2, 1/2)$ supersymmetry because Q_α commutes with P while $\bar{Q}_{\dot\alpha}$ does not. Deformations generated by a nilpotent Poisson structure such as (1.4) will be called *nilpotent deformations* in this paper.

Some quantum calculations in this deformed superspace, in particular concerning non-renormalization theorems, were presented in [9]–[12]. The string-theoretic origin of this type of noncommutativity was discussed in [13, 8, 10] (see also [1, 14, 15]). More recently, the issue of analogous deformations of $N = 2$ extended superspace in four dimensions was addressed in [10].

While the preservation of chirality is the fundamental underlying principle of $N = 1$ superfield theories [16], it is the power of *Grassmann harmonic analyticity* which replaces the use of chirality in $N = 2$ supersymmetric theories in four dimensions [17]–[20]. Therefore, it is natural to look for nilpotent deformations of $N = (1, 1)$ euclidean superspace which preserve this harmonic analyticity (perhaps in parallel with chirality). The basic aim of the present paper is to describe such deformations and to give deformed superfield actions for a few textbook examples of $N = 2$ theories. We analyze the role of the standard conjugation or an alternative pseudoconjugation in euclidean $N = 2$ supersymmetric theories and their deformations.

Section 2 generalizes Seiberg's simplest nilpotent deformation to euclidean $N = (1, 1)$ superspace. We call this a *chiral nilpotent deformation*. The corresponding bi-differential operator P acts on the left half of the $N = (1, 1)$ spinor coordinates only and preserves all chiralities. It retains $N = (1, 0)$ supersymmetry but generically breaks the R-symmetry SU(2) as well as the euclidean invariance $\mathrm{SO}(4) \to \mathrm{SU}(2)_R$. For judicious choices, however, $N = (1, 1/2)$ supersymmetry or the whole automorphism group $\mathrm{SO}(4) \times \mathrm{SU}(2)$ can be preserved.

In section 3 we reformulate the chiral nilpotent deformation in the euclidean version of $N = 2$ harmonic superspace [18]–[20] and show that it preserves Grassmann harmonic analyticity. In the analytic coordinates, the chiral deformation acts not only on the left-handed fermionic but also on the bosonic coordinates (albeit still in nilpotent fashion). This remains true when we restrict to the analytic subspace, i.e. consider Grassmann analytic superfields. As an interesting alternative, there exist nilpotent deformations which affect neither anti-chiral superfields nor analytic superfields but act in the central superspace. We call them *analytic nilpotent deformations*. In this case the analytic subspace is undeformed but chirality is no longer preserved, still leading to deformed products for general superfields e.g. in $N = (1, 1)$ super Yang-Mills. Remarkably, the full $N = (1, 1)$ supersymmetry remains intact here. This option does not exist in $N = 1$ superspace.

Noncommutative interactions of harmonic superfields are considered in section 4. Formally these interactions resemble those in the purely bose-deformed harmonic superspace of [5]. As an important difference though, the nilpotency of our deformations is expected to render quantum calculations much more feasible.

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The main novel developments in our work are the analysis of $N = 2$ supersymmetry breaking deformations in harmonic superspace and the construction of the relevant superfield models. The supersymmetry-preserving deformations of $N = 2$ superspace were considered in [2, 4, 5]. While writing this paper, a preprint [21] appeared which discusses supersymmetry-breaking deformations of $N = 2$ superspace on equal footing with supersymmetry-preserving ones.

2. Deformations of $N = 2$ chiral superspace

In the deformation (1.1) of $N = 1$, $D = 4$ euclidean superspace proposed in [8] one introduces non(anti)commutativity only for one half of the spinor coordinates. By construction, this deformation preserves the chiral representations of $N = 1$ supersymmetry. The bi-differential operator of [8] has the form (1.4) and acts on standard superfields $V(x_L^m, \theta^\alpha, \bar\theta^{\dot\alpha})$.

It is important to realize that the (pseudo)conjugation properties of spinors in 4D euclidean space with the group $\mathrm{Spin}(4) = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ are radically different from those in Minkowski space since left- and right-handed SU(2) spinors are independent. A conjugation or pseudoconjugation is defined as an antilinear map σ which acts in the algebra of complex superfields via $\sigma(AB) = \sigma(B)\,\sigma(A)$ and is reduced to the standar complex conjugation on complex numbers. Furthermore, a conjugation satisfies $\sigma\big(\sigma(A)\big) = A$ while a pseudoconjugation obeys $\sigma\big(\sigma(A)\big) = (-1)^{|A|}A$ where $|A|$ is some Z_2-grading of A. For $N = 1$ euclidean superspace to have the real dimension $(4|4)$ like its Minkowski counterpart, one is led to apply the following pseudoconjugation of the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ spinor Grassmann coordinates (see e.g. [22]):[2]

$$(\theta^\alpha)^* = \varepsilon_{\alpha\beta}\theta^\beta\,, \qquad (\bar\theta^{\dot\alpha})^* = \varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\beta}\,. \tag{2.1}$$

Here, the map * is a pseudoconjugation which squares to -1 on any odd θ monomial (and on the fermionic component fields) and to $+1$ on any even monomial (and on the bosonic component fields). So, when acting on bosonic fields, it is indistinguishable from the standard complex conjugation. It preserves the irreducible representations of the group Spin(4) (it is straightforward to check that (2.1) is consistent with the action of this group). Examples of bilinear real combinations of $N = 1$ euclidean spinors are

$$\mathrm{i}\,\theta\,\psi(x)\,, \qquad \mathrm{i}\,\theta\sigma^m\bar\theta\,, \qquad \mathrm{i}\,\psi(x)\sigma_m\partial_m\bar\psi(x) \tag{2.2}$$

where $\psi^\alpha(x)$ and $\bar\psi^{\dot\alpha}(x)$ are pseudoreal spinor fields (they satisfy the conditions (2.1)). As an important consequence of the pseudoreality of spinor coordinates, $N = 1$ euclidean chiral superfields can be chosen as real (like the general ones).

Though our main goal is to introduce consistent *nilpotent* deformations of $N = 2$ *harmonic* superspace, it is convenient to start the analysis in ordinary superspace. We consider euclidean $N = (1, 1)$ superspace and use the chiral coordinates

$$z_L \;\equiv\; (x_L^m, \theta_k^\alpha, \bar\theta^{\dot\alpha k}) \tag{2.3}$$

[2]We use the conventions $\varepsilon_{12} = -\varepsilon^{12} = \varepsilon_{\dot 1\dot 2} = -\varepsilon^{\dot 1\dot 2} = 1$ and $\sigma_m = (I, \mathrm{i}\vec\sigma)$ for the basic quantities in euclidean space.

which transform under $N = (1,1)$ supersymmetry as

$$\delta_\epsilon x^m_{\scriptscriptstyle L} = 2\mathrm{i}(\sigma^m)_{\alpha\dot\alpha}\theta^\alpha_k\bar\epsilon^{\dot\alpha k}\,, \qquad \delta_\epsilon\theta^\alpha_k = \epsilon^\alpha_k\,, \qquad \delta_\epsilon\bar\theta^{\dot\alpha k} = \bar\epsilon^{\dot\alpha k}\,, \tag{2.4}$$

where ϵ^α_k and $\bar\epsilon^{\dot\alpha k}$ are the transformation parameters. The 'central' bosonic coordinate x^m is related to the 'left' coordinate by

$$x^m_{\scriptscriptstyle L} = x^m + \mathrm{i}(\sigma^m)_{\alpha\dot\alpha}\theta^\alpha_k\bar\theta^{\dot\alpha k}\,. \tag{2.5}$$

As automorphisms we have the euclidean space spinor group Spin(4) and the R-symmetry group $\mathrm{SU}(2)\times\mathrm{O}(1,1)$ acting simultaneously on left and right spinors.

Let us dwell in some detail on the (pseudo)reality properties of $N = (1,1)$ superspace. We assume the Grassmann coordinates to be real with respect to the standard conjugation

$$\widetilde{\theta^\alpha_k} = \varepsilon^{kj}\varepsilon_{\alpha\beta}\theta^\beta_j\,, \qquad \widetilde{\bar\theta^{\dot\alpha k}} = -\varepsilon_{kj}\varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\beta j}\,, \qquad \widetilde{x^m_{\scriptscriptstyle L}} = x^m_{\scriptscriptstyle L}\,, \qquad \widetilde{AB} = \widetilde{B}\widetilde{A}\,. \tag{2.6}$$

This conjugation squares to the identity on any object, and with respect to it the $N = (1,1)$ superspace has the real dimension $(4|8)$. The component spinor fields enjoy the analogous conjugation properties. Eq. (2.6) is evidently compatible with both Spin(4) and R-symmetries, preserving any irreducible representation of these groups. However, the $N = (1/2, 1/2)$ superspace cannot be treated as a real subspace of the $N = (1,1)$ superspace if one considers only this standard conjugation.

Surprisingly, in the same euclidean $N = (1,1)$ superspace one can define an analog of the pseudoconjugation (2.1), namely

$$(\theta^\alpha_k)^* = \varepsilon_{\alpha\beta}\theta^\beta_k\,, \qquad (\bar\theta^{\dot\alpha k})^* = \varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\beta k}\,, \qquad (x^m_{\scriptscriptstyle L})^* = x^m_{\scriptscriptstyle L}\,, \qquad (AB)^* = B^*A^* \tag{2.7}$$

with respect to which the $N = (1/2, 1/2)$ superspace forms a real subspace. The existence of this pseudoconjugation does not imply any further restriction on the $N = (1,1)$ superspace. It preserves representations of $N = (1,1)$ supersymmetry and, like (2.1), is compatible with the action of the group Spin(4). It is also compatible with the R-symmetry group $\mathrm{O}(1,1)$. As for the R-symmetry group SU(2), it preserves only some U(1) subgroup of the latter. In other words, the standard conjugation (2.6) and the pseudoconjugation (2.7) act differently on objects transforming under non-trivial representations of this SU(2), e.g. on Grassmann coordinates. The map * squares to -1 on these coordinates and the associated spinor fields, and to $+1$ on any bosonic monomial or field. Yet, even on bosonic objects the two maps act in a different way if these objects belong to a non-trivial representation of the R-symmetry SU(2) (see eq. (3.3) below). Only on the singlets of the latter, e.g. scalar $N = (1,1)$ superfields and R-invariant differential operators, both maps act as the standard complex conjugation. In particular, the invariant actions are real with respect to both * and $^\sim$, despite the fact that the component fields may have different properties under these (pseudo)conjugations. Clearly, it is the pseudoconjugation * which is respected by the reduction $N = (1,1) \to N = (1/2, 1/2)$. Such a reduction preserves the pseudoreality but explicitly breaks the SU(2) R-symmetry.

In chiral coordinates, a *chiral nilpotent deformation* for products of superfields is determined by the following operator,

$$
\begin{aligned}
P &= -\frac{1}{2}\overleftarrow{\partial}{}^{k}_{\alpha}\, C^{\alpha\beta}_{kj}\overrightarrow{\partial}{}^{j}_{\beta} = -\frac{1}{2}\overleftarrow{Q}{}^{k}_{\alpha}\, C^{\alpha\beta}_{kj}\overrightarrow{Q}{}^{j}_{\beta} \qquad \text{such that} \\
APB &= -\frac{1}{2}(A\overleftarrow{\partial}{}^{k}_{\alpha})C^{\alpha\beta}_{kj}(\overrightarrow{\partial}{}^{j}_{\beta}B) = -\frac{1}{2}(-1)^{p(A)}(\partial^{k}_{\alpha}A)C^{\alpha\beta}_{kj}(\partial^{j}_{\beta}B) \\
&= -(-1)^{p(A)p(B)}BPA\,.
\end{aligned} \tag{2.8}
$$

Here, $C^{\alpha\beta}_{kj} = C^{\beta\alpha}_{jk}$ are some constants, $p(A)$ is the supersymmetry Z_2-grading, while $Q^k_\alpha = \partial^k_\alpha$ are the generators of left supersymmetry and the derivatives act as

$$
\partial^k_\alpha \theta^\beta_i = \delta^k_i\,\delta^\beta_\alpha \qquad \text{and} \qquad \bar{\partial}_{\dot{\alpha} i}\bar{\theta}^{\dot{\beta} k} = \delta^k_i\,\delta^{\dot{\beta}}_{\dot{\alpha}}\,. \tag{2.9}
$$

By definition, the operator (2.8) preserves both chirality and anti-chirality and does not touch the $\mathrm{SU}(2)_R$. It induces a graded Poisson bracket on superfields [2, 4]. We also demand P to be real, i.e. invariant under some antilinear map in the algebra of superfields. The two possible (pseudo)conjugations introduced above then lead to different conditions

$$
(2.7) \quad \Longrightarrow \quad (C^{\alpha\beta}_{kj})^* = C_{\alpha\beta kj} \tag{2.10}
$$

$$
(2.6) \quad \Longrightarrow \quad \widetilde{C^{\alpha\beta}_{kj}} = C^{kj}_{\alpha\beta}\,. \tag{2.11}
$$

Since $(APB)^* = B^*PA^*$ and $\widetilde{APB} = \widetilde{B}P\widetilde{A}$, our star-product satisfies the following natural rules:

$$
(A\star B)^* = B^*\star A^*\,, \qquad \widetilde{(A\star B)} = \widetilde{B}\star\widetilde{A}\,. \tag{2.12}
$$

Under $\mathrm{SU}(2)_L\times\mathrm{SU}(2)$, the constant deformation matrix C decomposes into a (3,3) and a (1,1) part (see also [21]),

$$
C^{\alpha\beta}_{kj} = C^{(\alpha\beta)}_{(kj)} + \varepsilon^{\alpha\beta}\varepsilon_{kj} I\,. \tag{2.13}
$$

It is worth pointing out that the (1,1) part preserves the full $\mathrm{SO}(4)\times\mathrm{SU}(2)$ symmetry. The (3,3) part may be diagonalized by employing $\mathrm{SU}(2)_L\times\mathrm{SU}(2)$ rotations, so that it can be brought to a minimal form of

$$
C^{(\alpha\beta)}_{(kj)} = \delta^{(\alpha\beta)}_{(kj)}\, C^{(\alpha\beta)}\,. \tag{2.14}
$$

Depending on the signs (or vanishing) of the constants $C^{(\alpha\beta)}$, after rescaling of θ^α_k this yields the Clifford algebra in three or in lower dimensions. In an operator realization, therefore, θ^α_k are proportional to Pauli matrices or fermionic creation/annihilation operators. This complements the bosonic oscillator representation of (non-nilpotently) noncommuting bosonic coordinates in their Darboux basis.

Note that the manifestly $N=2$ supersymmetric bi-differential operators of [2, 4] involve flat spinor derivatives D^k_α instead of partial derivatives. This choice violates chirality. In contrast, we basically follow the line of [8] and investigate deformations which preserve irreducible representations based on chirality and/or Grassmann harmonic analyticity (see section 3), but may explicitly break some fraction of supersymmetry.

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Given the operator (2.8), the Moyal product of two superfields reads

$$\begin{aligned} A\star B \,&=\, A\,\mathrm{e}^{P} B = A\,B + A\,P\,B + \frac{1}{2}A\,P^2 B + \frac{1}{6}A\,P^3 B + \frac{1}{24}A\,P^4 B \\ &= A\,B + \partial^k_\alpha N^\alpha_k(A,B,C)\,, \end{aligned} \tag{2.15}$$

where the identity $P^5=0$ was used and $N^\alpha_k(A,B,C)$ is some function of the superfields and the constants $C^{\alpha\beta}_{kj}$. By construction, this product is associative and satisfies the standard Leibniz rule

$$\partial_M(A\star B) = \partial_M A\star B + (-1)^{p(M)p(A)} A\star \partial_M B \tag{2.16}$$

for partial derivatives $\partial_M = (\partial^L_m, \partial^k_\alpha, \bar\partial_{\dot\alpha k})$ and equally for any differential operator not containing θ^α_k. In particular, this star product preserves antichirality, $D^k_\alpha \bar\Phi = 0$. From (2.15) it is easy to see that the chiral-superspace integral of the Moyal product of two superfields is not deformed,

$$\int \mathrm{d}^4x\,\mathrm{d}^4\theta\, A\star B = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\, A\,B\ , \tag{2.17}$$

while integrals of star products of three or more superfields are deformed. For the superspace coordinates we obtain the following graded star-product commutators,

$$\begin{aligned} &\{\theta^\alpha_k \stackrel{\star}{,} \theta^\beta_j\} = C^{\alpha\beta}_{kj}\,, \qquad &&[x^m_L \stackrel{\star}{,} x^n_L] = 0\,, \qquad &&[x^m_L \stackrel{\star}{,} \theta^\alpha_k] = 0\,, \\ &[x^m_L \stackrel{\star}{,} \bar\theta^{\dot\alpha k}] = 0 &&\{\theta^\alpha_k \stackrel{\star}{,} \bar\theta^{\dot\beta j}\} = 0\,, &&\{\bar\theta^{\dot\alpha k} \stackrel{\star}{,} \bar\theta^{\dot\beta j}\} = 0\,. \end{aligned} \tag{2.18}$$

The $N=(1,1)$ supersymmetry is realized in the standard way on all superfields, i.e. the supersymmetry transformations of the component fields are undeformed. At the same time, the star product (2.15) of two superfields is not a fully covariant object because the operator P of (2.8) breaks some of the symmetries. Thus, in our treatment only free actions preserve all supersymmetries while interactions get deformed and are not invariant under all standard supersymmetry transformations. To exhibit the residual symmetries of a deformed interacting theory, we formulate the invariance condition

$$[K,P]=0 \tag{2.19}$$

for the corresponding generators K in the standard $N=(1,1)$ superspace. Clearly, this condition is generically not met by differential operators depending on θ^α_k, such as

$$\begin{aligned} L^\alpha_\beta &= \frac{1}{2}(\sigma_{mn})^\alpha_\beta x^m_L \partial^n_L + \theta^\alpha_k \partial^k_\beta - \frac{1}{2}\delta^\alpha_\beta \theta^\rho_k \partial^k_\rho\,, \\ L^k_j &= \theta^\alpha_j \partial^k_\alpha - \frac{1}{2}\delta^k_j \theta^\rho_i \partial^i_\rho - \bar\theta^{\dot\alpha k}\bar\partial_{\dot\alpha j} + \frac{1}{2}\delta^k_j \bar\theta^{\dot\rho i}\bar\partial_{\dot\rho i}\,, \\ \bar Q_{\dot\alpha k} &= \bar\partial_{\dot\alpha k} - 2\mathrm{i}(\sigma^m)_{\alpha\dot\alpha}\theta^\alpha_k \partial^L_m\,, \end{aligned} \tag{2.20}$$

and the symmetries generated by these are explicitly broken in the deformed superspace integrals. Out of all supersymmetry and automorphism generators, only Q^k_α and $\bar L^{\dot\alpha}_{\dot\beta}$ do commute with P of (2.8). Hence, for a generic choice of the constant matrix $(C^{\alpha\beta}_{ik})$, the breaking pattern is $N=(1,1)\to N=(1,0)$ for supersymmetry and $\mathrm{SO}(4)\times\mathrm{O}(1,1)\times\mathrm{SU}(2)\to\mathrm{SU}(2)_R$ for euclidean and R-symmetries.

An exception occurs for the singlet part in (2.13), i.e. for

$$C^{(\alpha\beta)}_{(kj)} = 0 \quad \Longrightarrow \quad C^{\alpha\beta}_{kj} = \varepsilon^{\alpha\beta}\varepsilon_{kj} I \quad \Longleftrightarrow \quad P_{\text{singlet}} = -\frac{1}{2}\overleftarrow{Q^k_\alpha}\, I\, \overrightarrow{Q^\alpha_k}\,, \tag{2.21}$$

which is fully SO(4) $\times$ SU(2) invariant and non-degenerate but also fully breaks the right half of supersymmetry.

Is it possible to break less than one half of the supersymmetry? The answer depends on our choice of conjugation. The $\widetilde{\ }$ conjugation (2.6) connects Q^1_α with $Q^{\alpha 2}$, so the condition $\widetilde{P} = P$ necessarily breaks $N = (0,1)$ supersymmetry. The choice of the * pseudoconjugation (2.7), on the other hand, is compatible with the decomposition of $N = (1,1)$ into two $N = (1/2, 1/2)$ superalgebras, each given by a fixed value for the SU(2) index. Therefore, it allows one to pick a degenerate deformation, e.g.

$$P_{\text{deg}}(Q^2) = -\frac{1}{2} C^{12}_{22} (\overleftarrow{Q}{}^2_1\, \overrightarrow{Q}{}^2_2 + \overleftarrow{Q}{}^2_2\, \overrightarrow{Q}{}^2_1)\ , \tag{2.22}$$

which does not involve Q^1_α. In this case, only $\bar{Q}_{\dot{\alpha}2}$ are broken but not the supercharges $\bar{Q}_{\dot{\alpha}1}$. Hence, the deformation P_{deg} preserves $N = (1, \frac{1}{2})$ supersymmetry.

It is of course possible to consider more general deformations affecting both the chiral sector and the anti-chiral one. Since in euclidean space left and right sectors do not talk to each other, we may combine a chiral deformation P with some anti-chiral one, $\bar{P}$. The only (though rather restrictive) requirement is that $\bar{P}$ commutes with P. Otherwise, the important associativity property for the resulting deformation would be ruined. For instance, one may add to P of (2.8) any one of the following bi-differential operators,

$$L = -\frac{1}{2}\overleftarrow{D}{}^k_\alpha\, J\, \overrightarrow{D}{}^\alpha_k \qquad \text{or} \qquad R = -\frac{1}{2}\overleftarrow{\bar{D}}_{\dot{\alpha}k}\, \bar{J}\, \overrightarrow{\bar{D}}{}^{\dot{\alpha}k} = -\frac{1}{2}\overleftarrow{\bar{\partial}}_{\dot{\alpha}k}\, \bar{J}\, \overrightarrow{\bar{\partial}}{}^{\dot{\alpha}k}\ , \tag{2.23}$$

where J and $\bar{J}$ are some real constants. Each of these commutes with only one of the two chirality operators, but preserves the full supersymmetry and yields an S$|O(4) \times$ SU(2) invariant deformation. On their own, these operators produce deformations for general superfields without any symmetry breaking and so belong to the class of fully supersymmetric Moyal operators considered in [2, 4, 21]. The deformation operator $P{+}R$ or $P{+}L$ breaks the same portion of (super)symmetry as P does. It preserves chirality or anti-chirality, respectively. Since L and R induce graded star-commutators more general than the minimal set (2.18), the corresponding deformations lie outside the class of chiral nilpotent ones. We shall encounter them again in the following section.

3. Deformations of $N = 2$ harmonic superspace

The basic concepts of the $N = 2, D = 4$ harmonic superspace [18, 19] are collected in the book [20]. The spinor SU(2)/ U(1) harmonics $u^\pm_i$ subject to

$$u^{\pm k} = \varepsilon^{kj} u^\pm_j \qquad \text{and} \qquad u^{+k} u^-_k = 1 \qquad \text{with} \qquad \delta_\epsilon u^{\pm k} = 0 \tag{3.1}$$

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can be used to construct analytic coordinates $(x_{A}^{m}, \theta^{\pm\alpha}, \bar{\theta}^{\pm\dot{\alpha}})$ in the euclidean version of $N=2$ harmonic superspace:

$$
\begin{aligned}
x_{A}^{m} &= x_{L}^{m} - 2\mathrm{i}(\sigma^{m})_{\alpha\dot{\alpha}}\theta^{\alpha k}\bar{\theta}^{\dot{\alpha}j}u_{k}^{-}u_{j}^{+}\,, \qquad \delta_{\epsilon}x_{A}^{m} = -2\mathrm{i}(\sigma^{m})_{\alpha\dot{\alpha}}(\epsilon^{-\alpha}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\bar{\epsilon}^{-\dot{\alpha}})\,, \\
\theta^{\alpha\pm} &= \theta^{\alpha k}u_{k}^{\pm}\,, \qquad \bar{\theta}^{\pm\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}k}u_{k}^{\pm}\,, \qquad \delta_{\epsilon}\theta^{\alpha\pm} = \epsilon^{\pm\alpha}\,, \qquad \delta_{\epsilon}\bar{\theta}^{\pm\dot{\alpha}} = \bar{\epsilon}^{\pm\dot{\alpha}}\,,
\end{aligned}
\tag{3.2}
$$

where $\epsilon^{\pm\alpha} = \epsilon^{\alpha k}u_{k}^{\pm}$, $\bar{\epsilon}^{\pm\dot{\alpha}} = \bar{\epsilon}^{\dot{\alpha}k}u_{k}^{\pm}$, and $(x_{L}^{m}, \theta_{k}^{\alpha}, \bar{\theta}^{\dot{\alpha}k})$ are chiral coordinates of $N=(1,1)$ superspace. We extend the (pseudo)conjugations (2.6) and (2.7) to the harmonics by

$$
\widetilde{u_{k}^{\pm}} = u^{\pm k} \qquad \text{and} \qquad (u_{k}^{\pm})^{*} = u_{k}^{\pm}
\tag{3.3}
$$

so that the analytic coordinates conjugate identically for both choices,

$$
\begin{aligned}
\widetilde{x_{A}^{m}} &= x_{A}^{m}\,, \qquad \widetilde{\theta^{\pm\alpha}} = \varepsilon_{\alpha\beta}\theta^{\pm\beta}\,, \qquad \widetilde{\bar{\theta}^{\pm\dot{\alpha}}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\pm\dot{\alpha}}\,, \\
(x_{A}^{m})^{*} &= x_{A}^{m}\,, \qquad (\theta^{\pm\alpha})^{*} = \varepsilon_{\alpha\beta}\theta^{\pm\beta}\,, \; (\bar{\theta}^{\pm\dot{\alpha}})^{*} = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\pm\dot{\alpha}}\,;
\end{aligned}
\tag{3.4}
$$

in particular, both square to -1 on spinor coordinates. This means that both maps become pseudoconjugations when applied to the extended set of coordinates. These two pseudoconjugations act identically on invariants and harmonic superfields, e.g. $(A^{k}B_{k})^{*} = \widetilde{(A^{k}B_{k})}$ or $(q^{+})^{*} = \widetilde{q^{+}}$, but differ on harmonics or R-spinor component fields, e.g. $(A_{k})^{*} \neq \widetilde{A_{k}}$. An important invariant pseudoreal subspace is the analytic euclidean harmonic superspace, parametrized by the coordinates

$$
\zeta \equiv (x_{A}^{m}\ ,\ \theta^{+\alpha}\ ,\ \bar{\theta}^{+\dot{\alpha}}), u_{i}^{\pm}\,.
\tag{3.5}
$$

The supersymmetry-preserving spinor and harmonic derivatives have the following form in these coordinates ($D_{L}^{++} = \partial^{++}$):

$$
\begin{aligned}
D_{\alpha}^{+} &= \partial_{\alpha}^{+}\,, \qquad D_{\alpha}^{-} = -\partial_{\alpha}^{-} + 2\mathrm{i}(\sigma^{m})_{\alpha\dot{\alpha}}\bar{\theta}^{-\dot{\alpha}}\partial_{m}^{A}\,, \\
\bar{D}_{\dot{\alpha}}^{+} &= \bar{\partial}_{\dot{\alpha}}^{+}\,, \qquad \bar{D}_{\dot{\alpha}}^{-} = -\bar{\partial}_{\dot{\alpha}}^{-} - 2\mathrm{i}(\sigma^{m})_{\alpha\dot{\alpha}}\theta^{-\alpha}\partial_{m}^{A}\,,
\end{aligned}
\tag{3.6}
$$

$$
D_{A}^{++} = \partial^{++} - 2\mathrm{i}(\sigma^{m})_{\alpha\dot{\alpha}}\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{m}^{A} + \theta^{+\alpha}\partial_{\alpha}^{+} + \bar{\theta}^{+\dot{\alpha}}\bar{\partial}_{\dot{\alpha}}^{+}\,,
\tag{3.7}
$$

where $\partial_{\alpha}^{\pm} \equiv \partial/\partial\theta^{\mp\alpha}$, $\bar{\partial}_{\dot{\alpha}}^{\pm} \equiv \partial/\partial\bar{\theta}^{\mp\dot{\alpha}}$, $\partial^{++} = u^{+i}\partial/\partial u^{-i}$. The partial derivatives in different bases are related as

$$
\begin{aligned}
\partial_{m}^{L} &= \partial_{m}^{A}\,, \qquad D_{L}^{++} = \partial^{++} = D_{A}^{++}\,, \\
\partial_{\alpha}^{k} &= -u^{+k}\partial_{\alpha}^{-} - u^{-k}\partial_{\alpha}^{+} + 2\mathrm{i}u^{-k}\bar{\theta}^{+\dot{\alpha}}(\sigma^{m})_{\alpha\dot{\alpha}}\partial_{m}^{A}\,, \\
\bar{\partial}_{\dot{\alpha}k} &= u_{k}^{+}\bar{\partial}_{\dot{\alpha}}^{-} + u_{k}^{-}\bar{\partial}_{\dot{\alpha}}^{+} + 2\mathrm{i}u_{k}^{+}\theta^{-\alpha}(\sigma^{m})_{\alpha\dot{\alpha}}\partial_{m}^{A} = u_{k}^{-}\bar{D}_{\dot{\alpha}}^{+} - u_{k}^{+}\bar{D}_{\dot{\alpha}}^{-}\,.
\end{aligned}
\tag{3.8}
$$

A Grassmann analytic superfield is defined by

$$
D_{\alpha}^{+}\Phi(\zeta, \theta^{-}, \bar{\theta}^{-}, u) = \bar{D}_{\dot{\alpha}}^{+}\Phi(\zeta, \theta^{-}, \bar{\theta}^{-}, u) = 0
\tag{3.9}
$$

and so can be treated as an unconstrained function in the analytic superspace, $\Phi = \Phi(\zeta, u)$.

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It is important to realize that the chirality-preserving operator P in (2.8) also preserves *Grassmann analyticity*. This is seen in the analytic basis using the relations (3.8),

$$\{\partial^k_\alpha, D^+_\beta\} = \{\partial^k_\alpha, \bar{D}^+_{\dot\beta}\} = 0 \qquad \Longrightarrow \qquad [P, D^+_\beta] = [P, \bar{D}^+_{\dot\beta}] = 0\,. \tag{3.10}$$

In the analytic superspace it takes the form

$$P = -\frac{1}{2}\left[\overleftarrow{\partial^-_\alpha} u^{+k} - 2\mathrm{i}\overleftarrow{\partial^A_m} u^{-k}\bar\theta^{+\dot\alpha}(\sigma^m)_{\alpha\dot\alpha}\right] C^{\alpha\beta}_{kj}\left[u^{+j}\overrightarrow{\partial^-_\beta} - 2\mathrm{i}u^{-j}\bar\theta^{+\dot\beta}(\sigma^n)_{\beta\dot\beta}\overrightarrow{\partial^A_n}\right]. \tag{3.11}$$

We point out that our deformation is polynomial (see (2.15)), a property which may also be checked directly in harmonic superspace. The graded star-commutators in the analytic superspace read

$$\{\theta^{+\alpha} \overset{\star}{,} \theta^{+\beta}\} = C^{\alpha\beta}_{kj} u^{k+} u^{j+} =: \mathcal{C}^{++\alpha\beta}\,, \tag{3.12}$$

$$[x^m_A \overset{\star}{,} \theta^{+\alpha}] = 2\mathrm{i}(\sigma^m)_{\beta\dot\beta}\bar\theta^{+\dot\beta} C^{\beta\alpha}_{kj} u^{k-} u^{j+} =: \mathcal{C}^{m+\alpha}\,, \tag{3.13}$$

$$[x^m_A \overset{\star}{,} x^n_A] = 4(\sigma^m)_{\alpha\dot\alpha}(\sigma^n)_{\beta\dot\beta}\bar\theta^{+\dot\alpha}\bar\theta^{+\dot\beta} C^{\beta\alpha}_{kj} u^{k-} u^{j-} =: \mathcal{C}^{mn}\,. \tag{3.14}$$

Note that the functions $\mathcal{C}^{m+\alpha}$ and $\mathcal{C}^{mn}$ are nilpotent. For the singlet deformation (2.21) they simplify to

$$\mathcal{C}^{\alpha\beta++} = \mathcal{C}^{mn} = 0\,, \qquad \mathcal{C}^{m+\alpha} = -2\mathrm{i}(\sigma^m)^{\alpha\dot\beta}\bar\theta^+_{\dot\beta}\, I\,, \tag{3.15}$$

$$\Longrightarrow \qquad P_{\text{singlet}} = -\mathrm{i}(\sigma^m)^{\alpha\dot\beta}\bar\theta^+_{\dot\beta}\, I\,(\overleftarrow{\partial^A_m}\,\overrightarrow{\partial^-_\alpha} - \overleftarrow{\partial^-_\alpha}\,\overrightarrow{\partial^A_m}) \tag{3.16}$$

which satisfies $P^3_{\text{singlet}} = 0$. Since P is independent of harmonics, $[D^{\pm\pm}, P] = 0$, the deformed coordinate (anti)commutators are closed under the action of $D^{\pm\pm}$. The transformation properties of analytic superfields are standard like in the case of chiral superfields. As in the previous section, the deformation operator P generically breaks $N = (1,1) \to N = (1,0)$ supersymmetry and part of the euclidean and R symmetries. Note, however, that on analytic superfields the degenerate deformation $P_{\text{deg}}(Q^2)$ contains only

$$Q^2_\alpha = -u^{+2}\partial^-_\alpha + 2\mathrm{i}u^{-2}\bar\theta^{+\dot\alpha}(\sigma^m)_{\alpha\dot\alpha}\partial^A_m\,. \tag{3.17}$$

Hence, this deformation preserves chirality, analyticity and $N = (1, \frac{1}{2})$ supersymmetry.

If we do not care about chirality we may add to P any one of the two supersymmetry-preserving operators (2.23) which in analytic coordinates read

$$L = \frac{1}{2}\left(\overleftarrow{D}^{+\alpha} J \overrightarrow{D}^-_\alpha + \overleftarrow{D}^-_\alpha J \overrightarrow{D}^{+\alpha}\right), \qquad R = \frac{1}{2}\left(\overleftarrow{\bar D}^{+\dot\alpha} \bar J \overrightarrow{\bar D}^-_{\dot\alpha} + \overleftarrow{\bar D}^-_{\dot\alpha} \bar J \overrightarrow{\bar D}^{+\dot\alpha}\right). \tag{3.18}$$

These operators do not deform products of analytic superfields $\Phi(\zeta, u)$ and $\Lambda(\zeta, u)$:

$$\Phi\,\mathrm{e}^L \Lambda = \Phi\,\Lambda\,, \qquad \Phi\,\mathrm{e}^R \Lambda = \Phi\,\Lambda\,. \tag{3.19}$$

Our choice of deformation (2.8) was a minimal one for chiral superspace coordinates, in the sense that we took x^m_L and $\bar\theta^{\dot\alpha k}$ to star-(anti)commute with anything. Alternatively, one can conceive of minimal deformations for analytic superspace coordinates. For such *analytic nilpotent deformations*, the graded (anti)commutators of all analytic superspace coordinates $\big(x_A^m, \theta^{+\alpha}, \bar\theta^{+\dot\alpha}, u^{\pm i}\big)$ among themselves vanish, except perhaps for

$$\{\theta^{+\alpha} \overset{\star}{,} \theta^{+\beta}\} = \hat{C}^{++\alpha\beta} =: \hat{C}^{\alpha\beta}_{ik} u^{+i} u^{+k} \,, \tag{3.20}$$

where $\hat{C}^{\alpha\beta}_{ik}$ are constants. This does not yet fix the deformation since nothing was said about $\theta^{-\alpha}$ or $\bar\theta^{-\dot\alpha}$. It is the natural assumption that

$$D_L^{\pm\pm} x_L^m \equiv \partial^{\pm\pm} x_L^m = 0 \qquad \text{and} \qquad \partial^{\pm\pm}\theta^\alpha_k = \partial^{\pm\pm}\bar\theta^{\dot\alpha}_k = 0 \tag{3.21}$$

which uniquely determines the analytic nilpotent deformation. First, it maps (3.20) to

$$\{\theta^{+\alpha} \overset{\star}{,} \theta^{-\beta}\} = \hat{C}^{\alpha\beta}_{ik} u^{+i} u^{-k} \,. \tag{3.22}$$

Second, requiring

$$[x_A^m \overset{\star}{,} x_A^n] = 0 \qquad \text{and} \qquad [x_A^m \overset{\star}{,} \theta^{+\alpha}] = 0 \,, \tag{3.23}$$

relating x_A^m to x_L^m via (3.2) and using (3.22), the harmonic-independence assumption (3.21) straightforwardly yields $\hat{C}^{++\alpha\beta} = 0$, thus reducing $\hat{C}$ to its $\mathrm{SO}(4) \times \mathrm{SU}(2)$ singlet part, $\hat{C}^{\alpha\beta}_{ik} = \varepsilon^{\alpha\beta}\varepsilon_{ik} J$. Therefore, we arrive at

$$\{\theta^{+\alpha} \overset{\star}{,} \theta^{+\beta}\} = 0 \qquad \text{and} \qquad \{\theta^{+\alpha} \overset{\star}{,} \theta^{-\beta}\} = \varepsilon^{\alpha\beta}\, J \tag{3.24}$$

for this case. Third, the remaining graded (anti)commutators are easily reconstructed to be

$$\{\theta^{-\alpha} \overset{\star}{,} \theta^{-\beta}\} = 0 \qquad \text{and} \qquad [x_A^m \overset{\star}{,} \theta^-_\beta] = -2\mathrm{i}(\sigma^m)_{\beta\dot\beta}\bar\theta^{-\dot\beta}\, J \,. \tag{3.25}$$

Comparison with (3.18) shows that (3.23)–(3.25) are precisely and uniquely generated by the supersymmetry-preserving deformation operator L. To summarize, nilpotent analytic deformations correspond to L. At the same time, chirality is no longer preserved, since e.g.

$$[x_L^m \overset{\star}{,} \theta_{\beta i}] = -2\mathrm{i}(\sigma^m)_{\beta\dot\beta}\bar\theta_i^{\dot\beta}\, J \tag{3.26}$$

and x_L^m ceases to commute with itself as well.

Apparently, any theory which admits a formulation entirely in analytic harmonic superspace will stay undeformed under an analytic nilpotent deformation as defined above. This statement applies in particular to hypermultiplets. In contrast, theories which require the full $N = 2$ superspace for their off-shell formulation, e.g. gauge theories, will experience specific deformations.

Finally, we note that it is possible to define more general deformations in the analytic basis of $N = 2$ superspace if one denies the harmonic-independence conditions (3.21). Although such deformations are worth to be investigated, the deformations respecting $[D^{\pm\pm}, P] = 0$ are distinguished in that they retain a link with the standard $N = 2$ superspace. Note also that we do not analyze here deformations based on the SU(2) breaking Grassmann-analytic coordinates $\theta^{1\alpha}{+}\mathrm{i}\theta^{2\alpha}$ which were considered in [21].

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4. Interactions in deformed harmonic superspace

Harmonic superspace with noncommutative bosonic coordinates x_A^m has been discussed in [5]. This deformation yields nonlocal theories but preserves the whole $N=2$ supersymmetry. In contrast, we expect that the deformations defined in the previous section will produce much weaker nonlocalities due to their nilpotency. Leaving quantum considerations for future study, we present here the chirally deformed versions of the off-shell actions for some basic theories in harmonic superspace.

We shall limit our attention to the deformation operator P which affects analytic superfields and preserves both analyticity and chiralities. The free q^+ and ω hypermultiplet actions of ordinary harmonic theory [20] are not deformed in non(anti)commutative superspace:

$$S_0(q^+) = \int \mathrm{d}u\, \mathrm{d}\zeta^{-4}\ \tilde{q}^{\,+} D^{++} q^+\,, \qquad S_0(\omega) = \int \mathrm{d}u\, \mathrm{d}\zeta^{-4}\, (D^{++}\omega)^2\,, \tag{4.1}$$

where $\mathrm{d}\zeta = \mathrm{d}^4 x_A (D^-)^4$. Non(anti)commutativity arises in interactions, for instance for the self-interaction of the hypermultiplet,

$$S_4(q^+) = \int \mathrm{d}u\, \mathrm{d}\zeta^{-4}\, (a\ \tilde{q}^+ \star q^+ \star \tilde{q}^+ \star q^+ + b\ q^+ \star q^+ \star \tilde{q}^+ \star \tilde{q}^+)\,, \tag{4.2}$$

where a and b are real coupling constants. Expanding out the star products yields a finite number of corrections to the local interaction term $(q^+\tilde{q}^+)^2$. As an example, for the simplest nilpotent deformation (3.15) one has

$$\tilde{q}^+ \star q^+ = \tilde{q}^+ \left(1 + P_{\text{singlet}} + \frac{1}{2} P^2_{\text{singlet}}\right) q^+\,. \tag{4.3}$$

An mentioned before, these corrections explicitly break a part of the symmetries (2.20).

The interaction of the hypermultiplet q^+ with a U(1) analytic gauge superfield V^{++} can be introduced as in [5], by replacing D^{++} in (4.1) with the covariant harmonic noncommutative left-derivative,

$$D^{++} q^+ \qquad \Longrightarrow \qquad \nabla^{++} q^+ = D^{++} q^+ + V^{++} \star q^+\,. \tag{4.4}$$

The gauge transformation of the anti-hermitean V^{++} reads

$$\delta_\lambda V^{++} = -D^{++}\lambda + [\lambda \stackrel{\star}{,} V^{++}] \tag{4.5}$$

where λ is an anti-hermitean analytic gauge parameter. The generalization to U(n) analytic gauge fields is straightforward. Note again that from the beginning we retain only those symmetries which are unbroken by the deformation of choice.

In Wess-Zumino gauge we have

$$\begin{aligned} V^{++}_{WZ} = {} & (\theta^+)^2 \bar{\phi} + (\bar{\theta}^+)^2 \phi + \theta^{+\alpha} \bar{\theta}^{+\dot{\alpha}} A_{\alpha\dot{\alpha}} + (\theta^+)^2 \bar{\theta}^{+\dot{\alpha}} u^-_k \bar{\lambda}^k_{\dot{\alpha}} + \\ & + (\bar{\theta}^+)^2 \theta^{+\alpha} u^-_k \lambda^k_\alpha + (\theta^+)^2 (\bar{\theta}^+)^2 u^-_k u^-_j X^{kj}\,, \end{aligned} \tag{4.6}$$

with all components being functions of x_A^m, and a component expansion of the hypermultiplet q^+ which consists of infinitely many terms due to the harmonic dependence. The component expansion of the deformed products is rather complicated since the number of terms increases significantly. For the singlet deformation (3.15), the star product in (4.4) contains the terms

$$V^{++}P_{\text{singlet}}\, q^+ = \frac{1}{2}\mathcal{C}^{m\alpha+}[\partial_m^A V^{++}\partial_\alpha^- q^+ - \partial_\alpha^- V^{++}\partial_m^A q^+]\,, \tag{4.7}$$

$$V^{++}P^2_{\text{singlet}}\, q^+ = -\frac{1}{4}\mathcal{C}^{m\alpha+}\mathcal{C}^{n\beta+}\times \\ \times[\partial_m^A\partial_n^A V^{++}\partial_\alpha^-\partial_\beta^- q^+ + \partial_\alpha^-\partial_\beta^- V^{++}\partial_m^A\partial_n^A q^+ + 2\partial_\beta^-\partial_m^A V^{++}\partial_n^A\partial_\alpha^- q^+]\,. \tag{4.8}$$

Since $\mathcal{C}^{m+\alpha} = -2\mathrm{i}(\sigma^m)^{\alpha\dot\beta}\bar\theta^+_{\dot\beta} I$, all terms proportional to $(\bar\theta^+)^2$ in the component expansion of V^{++} or q^+ drop out of this deformation. In particular, the auxiliary field X_{kj} of the vector multiplet (4.6) does not appear. On the other hand, space-time derivatives of propagating fields do occur; for instance, the term $V^{++}P_{\text{singlet}}\, q^+$ produces

$$\mathrm{i}I(\sigma^m)_{\alpha\dot\beta}\bar\theta^{+\dot\beta}(\theta^+)^2\,\partial_m^A\bar\phi\,\psi^\alpha\,. \tag{4.9}$$

The action for this noncommutative U(1) gauge superfield can be constructed in central coordinates in analogy with the action for commutative $N=2$ Yang-Mills theory [23], but it is easier to analyze it in chiral coordinates. Following [23], one constructs the deformed connection for the derivative D^{--} via

$$D^{++}V^{--} - D^{--}V^{++} + [V^{++} \stackrel{\star}{,} V^{--}] = 0\,, \\ V^{--}(z_L,u) = \sum_{n=1}^{\infty}(-1)^n \times \\ \times\int \mathrm{d}u_1\cdots\mathrm{d}u_n\frac{V^{++}(z_L,u_1)\star V^{++}(z_L,u_2)\cdots\star V^{++}(z_L,u_n)}{(u^+u_1^+)(u_1^+u_2^+)\cdots(u_n^+u^+)}\,, \tag{4.10}$$

where $(u_1^+u_2^+)^{-1}$ is a harmonic distribution (see [19, 20]). In general, the action for V^{++} contains an infinite number of vertices, with star commutators substituting the ordinary commutators of V^{++} taken from the standard non-abelian action. On the cubic level, for example, one obtains

$$S_3(V^{++}) = \frac{1}{3g^2}\int \mathrm{d}u_1\,\mathrm{d}u_2\,\mathrm{d}u_3\,\mathrm{d}^4x_L\,\mathrm{d}^8\theta\,\frac{V^{++}(z_L,u_1)\star V^{++}(z_L,u_2)\star V^{++}(z_L,u_2)}{(u_1^+u_2^+)(u_2^+u_3^+)(u_3^+u_1^+)}\,, \tag{4.11}$$

where g is the gauge coupling. The chiral and anti-chiral superfield strengths $\mathcal{W}$ and $\bar{\mathcal{W}}$ in the euclidean case are independent. They have the form

$$\mathcal{W} = (\bar D^+)^2 V^{--}\,, \qquad \bar{\mathcal{W}} = (D^+)^2 V^{--}\,, \qquad \text{with} \qquad \delta_\lambda(\mathcal{W},\bar{\mathcal{W}}) = [\lambda \stackrel{\star}{,} (\mathcal{W},\bar{\mathcal{W}})]\,, \tag{4.12}$$

and satisfy the covariantized chirality and harmonic-independence conditions

$$\bar D^+_{\dot\alpha}\mathcal{W} = 0\,, \qquad \bar D^-_{\dot\alpha}\mathcal{W} - [\bar D^+_{\dot\alpha}V^{--} \stackrel{\star}{,} \mathcal{W}] = 0\,, \qquad D^{++}\mathcal{W} + [V^{++} \stackrel{\star}{,} \mathcal{W}] = 0\,, \tag{4.13}$$

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plus analogous conditions on the anti-chiral $\bar{\mathcal{W}}$, as well as $(D^+)^2\mathcal{W} = (\bar{D}^+)^2\bar{\mathcal{W}}$. For the case of the chirality-preserving deformations, one can write down gauge-invariant actions holomorphic in $\mathcal{W}$, such as

$$S_{\mathcal{W}} \sim \int \mathrm{d}^4x_L\, \mathrm{d}^4\theta \mathcal{W} \star \mathcal{W} \star \mathcal{W}\,, \tag{4.14}$$

and likewise for the covariant anti-chiral superfield strength. It is easy to check that

$$\delta_\lambda\, S_{\mathcal{W}} = 0 \qquad \text{and} \qquad D^{++}\, S_{\mathcal{W}} = \bar{D}_{\dot{\alpha}k}\, S_{\mathcal{W}} = 0\,. \tag{4.15}$$

In the Feynman rules, the only effect of our deformations is a small number of higher-derivative contributions to the standard interaction vertices. Due to the nilpotency of these deformations, the locality of the theory is not jeopardized. It should be straightforward to evaluate the ensueing mild corrections to the known quantum properties of $N = 2$ harmonic superspace.

5. Conclusions

We have considered nilpotent deformations of $N = (1, 1)$ chiral and Grassmann-analytic harmonic superfields. A minimal setup deforms only the fermionic coordinates and keeps the bosonic ones entirely commuting. Applied to chiral superspace coordinates, we call such a deformation *chiral nilpotent* because it preserves chirality. On the background of non-deformed euclidean $N = (1, 1)$ superspace, one can treat it as a soft breaking of the part of supersymmetry and automorphism symmetry which is generated by $\bar{Q}_{\dot{\alpha}k}$, L^α_β and L^k_j, respectively. Up to a choice of basis, the constant deformation matrix is determined by four parameters. Interestingly, a special choice of these retains the $\mathrm{SO}(4) \times \mathrm{SU}(2)$ automorphism invariance but still breaks $N = (1, 1) \to N = (1, 0)$ supersymmetry. Yet, contrary to an assertion of a recent preprint [24], it is possible to keep a fraction of $\frac{3}{4}$, i.e. $N = (1, \frac{1}{2})$ supersymmetry, by employing a *degenerate* deformation matrix. Complete supersymmetry, however, can only be saved at the expense of chirality.

As the main new development, we extended the analysis to euclidean $N = (1, 1)$ *harmonic* superspace, parametrized by analytic coordinates. For these, chiral nilpotent deformations induce nilpotent noncommutativity also in the bosonic sector but preserve Grassmann analyticity. It is therefore consistent to deform only the analytic subspace in this manner. As an alternative, we also investigated the minimal situation in analytic coordinates. Such *analytic nilpotent* deformations may respect full supersymmetry but violate chirality. It turned out that they do not affect the analytic subspace but only the central superspace; hence, they leave hypermultiplets undeformed. Finally, we gave examples of superfield theories in chiral-nilpotently deformed harmonic superspace. In particular, we have shown how to construct the $\mathrm{SO}(4) \times \mathrm{SU}(2)$ invariant nilpotent deformation of $N = (1, 1)$ supersymmetric U(1) gauge theory in chiral superspace coordinates.

It would be interesting to understand a possible string theoretic origin of the deformations considered here. On the more technical side, one may work out the effect of these deformations on the component actions and perform quantum calculations, e.g. for obtaining effective actions, in the off-shell harmonic superspace approach.

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Note added. Soon after the first version of our paper was listed on hep-th, there appeared an e-print [25] also addressing nilpotent singlet deformations of $N = 2$ harmonic superspace and construction of the corresponding field models.

Acknowledgments

This work was partially supported by the DFG priority program SPP 1096 in string theory, INTAS grant No 00-00254, RFBR-DFG grant No 02-02-04002, grant DFG No 436 RUS 113/669, RFBR grant No 03-02-17440 and a grant of the Heisenberg-Landau program.

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